High School Calculus in Focus: A Comparative Study of the Intended Curricula of Select Countries

Harold B. Badilla
University of the Philippines Integrated School
hbbadilla@up.edu.ph

Abstract

While many countries are integrating calculus into their high school curricula, there is limited research on how these curricula differ among nations with varying levels of mathematical achievement. To address this gap, this study compared the intended high school calculus curriculum of four countries: the Philippines, South Africa, New Zealand, and Ireland. Through curriculum mapping, the research identified similarities and differences in topic coverage and learning progression across these curricula. The results revealed that the Philippines and New Zealand's Level 8 achievement standards covered the most topics, while South Africa and Ireland's ordinary level curriculum covered the fewest. The Philippines, New Zealand's Level 8 achievement standards and Ireland's higher level curriculum were well-aligned, rigorously covering derivatives and integrals of both algebraic and transcendental functions. In contrast, South Africa, New Zealand's level 7 achievement standards, and Ireland's ordinary level curriculum focused more narrowly on algebraic functions. Notably, none of the countries defined limits using the ε-δ definition, and only the Philippines formally defined the definite integral. Structurally, only the Philippines treated calculus as a separate subject, while New Zealand adopted a unique spiral approach to teaching calculus topics. These findings suggest the need for further investigation into how these curriculum differences impact students' understanding of calculus.

Keywords: High School Calculus, Curriculum Mapping, Intended Curriculum

INTRODUCTION

In recent centuries, a significant shift in mathematics education has been witnessed across different educational systems. Calculus, a subject usually encountered for the first time at the university level, was incorporated into the high school curricula of various countries (Zuccheri & Zudini, 2014). Notable mathematicians pushed for this development in their wish to close the gap between high school and university-level calculus (Törner et al., 2014). Further recognizing its importance in preparing students for advanced science, technology, engineering, and mathematics (STEM) studies, education policymakers established calculus as one of the crucial components of secondary education.

Ayebo et al. (2016) revealed that success in college calculus and succeeding math courses is significantly associated with extensive high school calculus preparation. According to the study, introducing students to preliminary calculus concepts and notations prepares them for the rigors of college-level calculus. Reports also suggest that in countries whose study of high school calculus is optional (e.g., in the United States), more students are taking high school calculus because it undeniably boosts their college applications (Jaschik, 2022).

However, the inclusion of calculus in the high school curriculum has also faced backlash. Critics argue that high schools should focus on developing a solid foundation in calculus prerequisites like algebra, geometry, and trigonometry, as mastery in these subjects has double the impact on college calculus preparation than taking high school calculus (Sadler & Sonnert, 2018). Similarly, a report by Knudson (2015) said that “rushing” students to go through high school calculus shortchanges them of preparation to succeed in subjects beyond calculus.

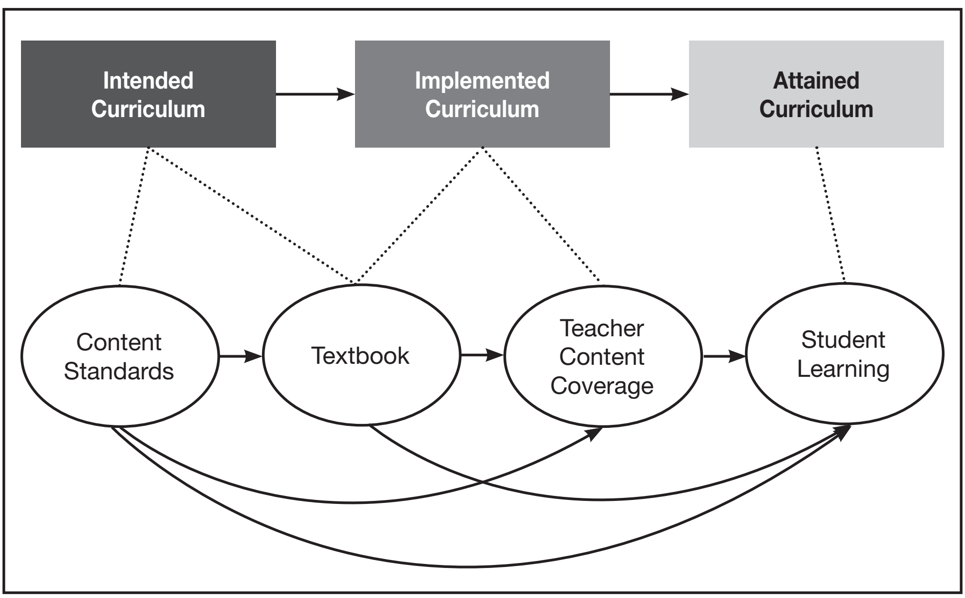
Nonetheless, the adoption of calculus in high school curricula remains a trend in many countries. As a result, different countries have been implementing varying curricula whose breadth and depth of topics have been diverse. While there is a substantial number of research in calculus education, commonly covering the students (e.g., their difficulties and misconceptions), teachers (e.g., their knowledge, beliefs, and practices) as well as the learning setups (e.g., pedagogical and curricular innovations) (Rasmussen et al., 2014), there is a little research concerning the nuanced differences in curriculum content and depth in high school calculus among different countries.

Consequently, this paper aims to perform a comparative analysis of high school calculus intended curricula in the Philippines, South Africa, New Zealand, and Ireland. Programme for International Student Assessment (PISA) 2022 report in Mathematics indicates that Ireland and New Zealand scored above the Organization for Economic Cooperation and Development (OECD) average, with Ireland ranking 11th and New Zealand ranking 23rd among the 81 participating countries/economies (OECD, 2023). Meanwhile, the Philippines and South Africa are among the least-performing countries in recent large-scale international mathematics assessments, with the Philippines placing 77th among the 81 participating economies in PISA 2022 (Ines, 2023), while South Africa placed 38th out of the 39 participating economies in Trends in International Mathematics and Science Study (TIMSS) 2019 report for eighth-grade mathematics (Mullis et al., 2020). Consequently, given that their official mathematics curricula are accessible and written in English, this research investigates the similarities and differences in terms of coverage and learning progression of topics in high school calculus among these countries, whose mathematics achievement varies as measured by TIMSS and PISA. This study assesses the structure of calculus as well as highlight best practices in crafting curriculum that could offer potential policy recommendations.

Intended Curriculum and Its Key Elements

Among its varied definitions, Tyler (1957, as cited in Glatthorn et al., 2019) defined curriculum as planned learning experiences designed for students to achieve specific educational goals. It serves as the "skeleton" of education, shaping and directing instruction (Houang & Schmidt, 2008). Three of its types are intended, implemented, and attained curricula (Borji & Farsani, 2023). The intended curriculum outlines the objectives that policymakers set for students, the implemented curriculum refers to how teachers deliver it (Bilbao et al., 2015; Hirsch & Reys, 2009), and the attained curriculum reflects what students learned (Wang & McDougall, 2018). The interplay among these three forms of curriculum is shown in Figure 1.

Figure 1
Relationship of Intended, Implemented, and Attained Curricula
(Wang & McDougall, 2018)



Empirical links among the elements in the model are indicated by solid lines, while definitional links are shown by dashed lines. Evidently, the implemented and attained curricula are influenced by the intended curriculum. The content standards specified in the intended curriculum significantly impact the teacher's content coverage and student learning. Textbooks can be seen as a potentially implemented curriculum, linking the intended topics to the classroom content (Schmidt et al., 2001).

An examination of the intended curriculum can be done by looking at the curriculum's coverage and learning progression. The curriculum's coverage is indicated by the number of topics covered at all grade levels (Wang & McDougall, 2018) while learning progression outlines the content and skills that students have to master (Borji & Farsani, 2023).

Research on High School Calculus Curriculum

Several papers investigated the high school calculus curriculum worldwide. Törner et al. (2014) found that France, Germany, the United Kingdom (England), Belgium, Italy, Greece, and Cyprus generally cover the basics of limits, derivatives, and integrals in different grade levels, whose covered topics vary depending on the student's chosen track. They primarily focus on procedures rather than concepts, which is also echoed in the study of Thomas et al. (2015). Bressoud et al. (2016) reported similar findings in South Korea, Germany, Uruguay, Hong Kong, the United States and France, focusing more on the application of topics than theorem proofs.

On the other hand, Ghedamsi and Lecorre (2021) revealed that Tunisia's curriculum provides accurate definitions of sequence convergence and integrals as well as formal proofs of theorems. Yoon et al. (2021) examined the curriculum changes in South Korea, noting the reduction of calculus topics due to stakeholders' concern about students' heavy workload. The limits of sequences were no longer covered at the beginning, and the definite integral was illustrated using the Fundamental Theorem of Calculus Part II rather than defining it as a limit of the Riemann sum, as it was before. The discussion of the limits of sequences would only begin if the student took Calculus after their Mathematics II.

Meanwhile, the Korea Institute for Curriculum and Evaluation and New Zealand Qualifications Authority (2015) compared the high school calculus curricula of New Zealand and South Korea. In the findings, it was reported that most of the topics in Korean Calculus I and II are also found in New Zealand's Level

8 achievement standards, except for convergence and divergence of sequences and series as well as a volume of a 3D diagram being present in South Korea.

In these studies cited, it is noted that the comparison of calculus curriculum in high schools focused on the countries performing above average in international assessments, mostly by Western countries with the exception of a few East Asian countries which consistently ranked among the best in mathematics. Consequently, this research aims to provide an account of the countries that are yet to be primarily studied in high school calculus. The countries chosen for this study are diverse in terms of history, culture, educational contexts, and ranking in international assessments, where comparative study on high school calculus curricula is lacking.

Educational Contexts of the Philippines, South Africa, New Zealand, and Ireland

The four countries in this study have different educational contexts regarding when and how high school calculus is taught to students. In the Philippines, calculus is studied in a semester in their first year in senior high school (Grade 11) should the student decide to take the STEM track (Department of Education, n.d.). In South Africa, students can learn the basics of differential calculus in their last year in high school (Grade 12) should they choose the subject mathematics in their chosen pathway in upper secondary education, known as Further Education and Training (FET) (Department of Basic Education, 2012). In New Zealand, calculus is taught under the mathematics subject during their senior secondary education, specifically under their curriculum's level 7 and 8 achievement standards (Ministry of Education, 2014).

Meanwhile, in Ireland, their upper secondary education, known as the Senior Cycle, offers the Leaving Certificate (Established) program, where five or more subjects are chosen by the students for summative examination at the cycle's end (National Council for Curriculum and Assessment, 2024). Among these subjects is mathematics, which students can study at three levels (foundation, ordinary, and higher) depending on their choice (O'Meara et al., 2023). It is under the ordinary and higher levels of mathematics in the Senior Cycle that students can learn calculus.

Among these countries, it is worth noting that the Philippines and South Africa adopt a highly centralized educational system (Bertram et al., 2021; Philippine Business for Education, n.d.). Following this top-down approach, the curriculum and teaching guides from their education departments and teachers across the

© 2024 | ISSN 2546-0730 (Online) | ISSN 1908-8701 (Print)

country are expected to carry out the plan laid out by the agencies (Bernardo & Garcia, 2006). This setup is also observed in Ireland where the Department of Education Skills develops the curriculum following the National Council for Curriculum and Assessment (NCCA) recommendation (Mullis et al., 2016). On the other hand, New Zealand adopts a decentralized educational system (Mullis et al., 2016). New Zealand Qualifications Authority & Korea Institute for Curriculum and Evaluation (2015) report that New Zealand gives schools more flexibility in designing learning activities for students, particularly at senior secondary schools where students have diverse learning pathways.

METHODOLOGY

To compare the high school calculus topics taught in the Philippines, South Africa, New Zealand, and Ireland, curriculum mapping was utilized as outlined in Greatorex et al. (2019). The following intended curricula were analyzed: the Philippines' Basic Calculus curriculum guide (Department of Education, 2016), South Africa's curriculum and assessment policy statement in mathematics for grades 10-12 (Department of Basic Education, 2011), New Zealand's achievement objectives for mathematics (Ministry of Education, 2014), and Ireland's Leaving Certificate mathematics syllabus for the senior cycle (National Council for Curriculum and Assessment, 2015). To ascertain the equivalence of topics in these curricula, Table 1 shows the books used as supplementary sources. These books are either state-owned or among the widely-used and best-selling textbooks in their countries.

Table 1
High school calculus textbooks used in the Philippines, South Africa, New Zealand, and Ireland

Philippines	Balmaceda, J. M., Vergara, T. H., De Lara, M. L., Sato, R., & Borja, G. J. (2016). Basic Calculus Learner's Material (1st ed.). Department of Education. (State-owned)
South Africa	Jenkin, A., van Zyl, M., & Kannemeyer, L. (n.d.). Everything Maths: Grade 12 Mathematics. Siyavula Education. (State-owned)
New Zealand	Level 7 Achievement Standards: Lakeland, R., & Nugent, C. (n.d.). EAS 2.7: Year 12 Mathematics and Statistics. Nulake Ltd. Level 8 Achievement Standards: Lakeland, R., & Nugent, C. (n.d.-b). Year 13 EAS Calculus Workbook. Nulake Ltd.

Ireland	Ordinary Level: Keating, M., McElroy, J., Mulvany, D., Murphy, O., O'Loughlin, J., & Townsend, C. (2017). Active Maths 3 (2nd ed.). Folens Publishers. Higher Level: Keating, M., McElroy, J., Mulvany, D., O'Loughlin, J., & Townsend, C. (2016). Active Maths 4 (2nd ed.). Folens Publishers.

The calculus topics from the curricula of the Philippines, South Africa, New Zealand's levels 7 and 8 achievement standards, and Ireland's ordinary and higher levels were coded and aligned with the 31 topics identified in the study using a table (Wang & McDougall, 2018). If a topic is included in the intended curriculum of the countries, it is placed in a cell where the column for the country's curriculum intersects with the row of the equivalent topic. This table was used to compare the topic coverage of high school calculus in each country. Following this, data analysis was performed to determine the similarities and differences among the countries in terms of coverage and learning progression (Borji & Farsani, 2023).

RESULTS AND DISCUSSION

The countries' topic coverage and learning progression in high school calculus are discussed in these sections.

Topic Coverage in the Intended Curricula

Table 2 shows the topic coverage of high school calculus in the Philippines, South Africa, New Zealand (levels 7 and 8 achievement standards), and Ireland (ordinary and higher levels). The Philippines and South Africa cover 26 and 9 out of the 31 topics, respectively. New Zealand's levels 7 and 8 achievement standards cover 11 and 26 topics, while Ireland's ordinary and higher levels cover 9 and 22 topics, respectively.

 | ISSN 2546-0730 (Online) | ISSN 1908-8701 (Print)

Table 2
Topic coverage of high school calculus in the Philippines, South Africa, New Zealand, and Ireland

Chapters	Philippines	South Africa	New Zealand		Ireland		Frequency
			Level 7	Level 8	Ordinary Level	Higher Level	
Limits and Continuity	Limit of a Function	Limit of a Function		Limit of a Function	Limit of a Function	Limit of a Function	5
	Limit Theorems and Examples Involving Algebraic and Transcendental Functions					Limit Theorems and Examples Involving Algebraic Functions	2
	Some Special Limits, Limits of Indeterminate Forms	Limits of Indeterminate Forms		Limits of Indeterminate Forms	Limits of Indeterminate Forms	Limits of Indeterminate Forms	5
						Limits of Sequences	1
	Continuity at a Point and on an Interval			Continuity on an Interval		Continuity at a Point	3
	Types of Discontinuities			Identifying Discontinuities			2

Chapters	Philippines	South Africa	New Zealand		Ireland		Frequency
			Level 7	Level 8	Ordinary Level	Higher Level	
Limits and Continuity	Intermediate Value and Extreme Value Theorems and Word Problems Involving Continuity						1
Derivatives and Their Applications	Tangent and Normal Lines to the Graph of a Function at a Point and Their Equations	Tangent and Normal Lines to the Graph of a Function at a Point and Their Equations	Tangent Line to the Graph of a Function at a Point and Its Equation	Tangent and Normal Lines to the Graph of a Function at a Point and Their Equations	Tangent Line to the Graph of a Function at a Point and Its Equation	Tangent Line to the Graph of a Function at a Point and Its Equation	6
	Formal Definition of Derivative	Formal Definition of Derivative	Informal Definition of Derivative	Formal Definition of Derivative	Informal Definition of Derivative	Formal Definition of Derivative	6
	Differentiability and Continuity			Differentiability and Continuity			2
	Differentiation Rules and Examples Involving Algebraic and Transcendental Functions	Differentiation Rules (Product and Quotient Rules excluded), and Examples Involving Algebraic Functions	Power Rule, Derivative of Algebraic Functions	Differentiation Rules and Examples Involving Algebraic and Transcendental Functions	Differentiation Rules (Product and Quotient Rules excluded) and Examples Involving Linear, Quadratic and Cubic Functions	Differentiation Rules and Examples Involving Algebraic and Transcen-dental Functions	6

© 2024 I ISSN 2546-0730 (Online) I ISSN 1908-8701 (Print)

Chapters	Philippines	South Africa	New Zealand		Ireland		Frequency
			Level 7	Level 8	Ordinary Level	Higher Level	
Derivatives and Their Applications	Higher Order Derivatives	Second Derivative		Second Derivative	Second Derivative	Second Derivative	5
	Chain Rule			Chain Rule		Chain Rule	3
	Implicit Differentiation			Implicit Differentiation		Implicit Differentiation	3
				Parametric Equations and their Derivatives			1
	Optimization Problems	Optimization Problems	Optimization Problems	Optimization Problems	Optimization Problems	Optimization Problems	6
	Related Rates Problem			Related Rates Problem		Related Rates Problem	3
	Rates of Change, Distance, Velocity and Acceleration Problems	Rates of Change, Distance, Velocity and Acceleration Problems	Rates of Change, Distance, Velocity and Acceleration Problems	Rates of Change, Distance, Velocity and Acceleration Problems	Rates of Change, Distance, Velocity and Acceleration Problems	Rates of Change, Distance, Velocity and Acceleration Problems	6
		Application of Derivatives in Curve Sketching	Application of Derivatives in Curve Sketching	Application of Derivatives in Curve Sketching	Application of Derivatives in Curve Sketching	Application of Derivatives in Curve Sketching	5
Integrals and Their Applications	Illustration of an Antiderivative of a Function		Illustration of an Antiderivative of a Function	Illustration of an Antiderivative of a Function		Illustration of an Antiderivative of a Function	4

Chapters	Philippines	South Africa	New Zealand		Ireland		Frequency
			Level 7	Level 8	Ordinary Level	Higher Level	
Integrals and Their Applications	Antiderivatives of Algebraic and Transcendental Functions		Antiderivatives of Algebraic Functions	Antiderivatives of Algebraic and Transcendental Functions		Antideriva-tives of Algebraic and Transcendental Functions	4
	Antidifferentiation by Substitution			Antidifferentiation by Substitution			2
	Solving Differential Equations and Its Applications			Solving Differential Equations and Its Applications		Solutions to Differential Equations and Its Applications	3
	Approximation of Area using Riemann Sums		Approximation of Area using Rectangular and Trapezoidal Rules	Approximation of Area using the Trapezoidal and Simpson's Rules		Approximation of Area using Rectangular and Trapezoidal Rules	4
	Formal Definition of Definite Integral						1
	Fundamental Theorem of Calculus		Fundamental Theorem of Calculus	Fundamental Theorem of Calculus		Fundamental Theorem of Calculus	4
	Substitution Rule for Definite Integrals			Substitution Rule for Definite Integrals			2

Chapters	Philippines	South Africa	New Zealand		Ireland		Frequency
			Level 7	Level 8	Ordinary Level	Higher Level	
Integrals and Their Applications	Areas of Plane Regions Using Definite Integrals		Areas of Plane Regions Using Definite Integrals	Areas of Plane Regions Using Definite Integrals		Areas of Plane Regions Using Definite Integrals	4
	Application of Definite Integrals: Word Problems			Application of Definite Integrals: Word Problems			2
				Finding Volumes using Integration			1
						Average Value of a Function	1
31 topics	26	9	11	26	9	22	

Most of the countries' curricula begin the discussion of the limit of a function using a table of values and graphs, except for New Zealand's level 7 achievement standards, which do not cover limits, and Ireland's higher level, which follows a distinct sequence of topics. Additionally, these countries cover limits of indeterminate forms, usually solved using factoring. Only the Philippines, New Zealand's level 8 achievement standards, and Ireland's higher level include the topic of continuity.

Among the chapters, derivatives and their applications are the most widely covered by the countries in this study. Most curricula cover the tangent line to the graph of a function and its equation to establish the definition of a derivative. In Ireland's ordinary level, however, the rules of differentiation are taught before associating derivatives with the slope of the tangent line to a curve. All the countries discuss various differentiation rules and their applications in optimization and kinematics.

Finally, only the Philippines, New Zealand (both levels 7 and 8 achievement standards), and Ireland's higher level include integrals and their applications in their high school calculus curriculum. Common topics among these countries include the illustration of antidifferentiation, particularly of algebraic functions, the approximation of the area under the curve using Riemann sums, the connection between differentiation and integration via the Fundamental Theorem of Calculus, and the computation of the area of plane regions using definite integrals. Additionally, the topic of finding general and particular solutions to differential equations and their applications is covered by these countries, except for New Zealand's level 7 achievement standards. It is also worth noting that the topic of differential equations is included in the differentiation chapter of Ireland's higher level, as its focus is simply on verifying solutions to differential equations.

Learning Progression in the Intended Curricula

This section will discuss the expected student knowledge and skills that constitute the learning progression, organized by chapter.

Limits and Continuity

The Philippines expects its students to develop a substantial understanding of limits and continuity. After finding limits using tables of values and graphs, students are expected to calculate the limits of algebraic and transcendental functions using limit theorems. The curriculum also expects students to solve problems involving continuity by first understanding continuity at a point and on an interval, types of discontinuities, and various theorems related to continuity. However, students in the Philippines are not expected to identify the limits of sequences.

South Africa and Ireland's ordinary level curricula share similar expectations for their students in this chapter. They briefly cover limits, expecting students to gain an intuitive understanding of limits through tables of values and graphs, followed by the calculation of limits of indeterminate forms. Neither curriculum includes a discussion on continuity.

While New Zealand's level 7 achievement standards do not cover limits and continuity, these concepts are addressed in the level 8 achievement standards. In addition to the common topics covered by the other countries, New Zealand's level 8 achievement standards informally define that a function is continuous on an interval *(a, b)* "if its graph can be drawn from $x = a$ to $x = b$

© 2024 | ISSN 2546-0730 (Online) | ISSN 1908-8701 (Print)

without lifting the pen from the paper." Furthermore, students are expected to identify discontinuities simply by examining the function's graph.

Lastly, Ireland's higher level curriculum has a distinct approach to introducing limits and continuity compared to the other countries in this study. While the other countries begin introducing limits using tables of values and graphs, Ireland's higher level introduces limits earlier in the sequences and series chapter. After covering arithmetic and geometric sequences and series, students are expected to evaluate limits of sequences and to determine whether a sequence is convergent or divergent, primarily dealing with limits at infinity. The curriculum covers most of the topics in this chapter except for types of discontinuity, the intermediate value and extreme value theorems, and word problems involving continuity.

Derivatives and Their Applications

The Philippines and Ireland's higher level curricula aim for students to develop significant skills in this chapter. Both curricula envision students differentiating algebraic and transcendental functions, unlike other countries that limit their examples to algebraic functions. They are also among the few curricula that expect students to apply the chain rule and implicit differentiation. However, while Ireland's higher level includes all topics related to the applications of derivatives, the Philippines does not cover the application of derivatives in curve sketching. On the other hand, Ireland's higher level does not connect continuity with differentiability.

Similar to their approach to limits, South Africa and Ireland's ordinary level curricula share similar expectations in this chapter. Both primarily focus on differential calculus, beginning with students understanding and finding the equation of the tangent line to a function, leading to the introduction of derivatives. However, while South Africa expects its students to formally define derivatives using its limit definition, Ireland's ordinary level only requires an informal definition, viewing the derivative as the slope of the tangent line to a curve at any point. Both curricula expect students to find derivatives using differentiation rules, except for the product and quotient rules, and they primarily focus on differentiating polynomials. Their curricula do not cover the chain rule or implicit differentiation, which are necessary for differentiating composite and more complex functions. Nonetheless, they expect students to solve problems involving the applications of derivatives in optimization, kinematics, and curve sketching, excluding related rates problems.

Meanwhile, New Zealand's level 7 achievement standards introduce calculus by having students understand the gradient of a curve, eventually leading to an informal definition of differentiation as the process of finding the gradient function. Since calculus at this level is introductory, the differentiation rule that students focus on is the power rule for differentiating polynomials, the most common type of function covered at this level. Nevertheless, like South Africa, New Zealand expects its students to solve problems in optimization, rates of change, motion, and curve sketching to demonstrate various applications of differentiation. For its level 8 achievement standards, New Zealand envisions students defining derivatives formally, establishing the relationship between differentiability and continuity, and differentiating transcendental functions using differentiation rules, including the chain rule and implicit differentiation. Additionally, New Zealand is the only country that expects students to work with parametric equations and their derivatives in its intended curriculum.

Integrals and Their Applications

The Philippines, New Zealand, and Ireland have some distinct expectations in this chapter. While all the countries that cover this chapter expect students to approximate the area under a curve using Riemann sums, the Philippines is the only country in this study that envisions students formally defining the definite integral as the limit of a Riemann sum. On the other hand, only New Zealand's level 8 achievement standards and the Philippines expect students to perform antidifferentiation by substitution, understand the substitution rule for definite integrals, and solve practical word problems involving definite integrals, particularly those involving the area under a curve. Additionally, New Zealand's level 8 achievement standards expect students to find volumes as another application of integration, while Ireland's higher level includes finding the average value of a function using integration.

CONCLUSION

In summary, this study provides a comprehensive analysis of the intended high school calculus curriculum in the Philippines, South Africa, New Zealand, and Ireland. It compared the topic coverage and learning progression of high school calculus in these countries.

In terms of topic coverage, the Philippines and New Zealand's Level 8 achievement standards include the highest number of calculus topics, covering 26 in total out of the 31 topics, while South Africa and Ireland's ordinary level

© 2024 | ISSN 2546-0730 (Online) | ISSN 1908-8701 (Print)

curricula cover the fewest, with 9 topics each. New Zealand's Level 7 achievement standards include 11 topics, while Ireland's higher level curriculum covers 22 topics. The Philippines, New Zealand's level 8 achievement standards, and Ireland's higher level curriculum generally align well, both aiming to cover high school calculus rigorously, including derivatives and integrals of both algebraic and transcendental functions. On the other hand, South Africa and Ireland's ordinary level curricula align closely, covering nearly identical topics, touching briefly on limits and focusing primarily on the derivatives of algebraic functions. New Zealand's Level 7 achievement standards focus on preliminary concepts of derivatives and integrals, while its Level 8 standards are notable for their unique topics, such as differentiating parametric equations and finding volumes through integration.

Among the topics covered, none of the countries in this study define the limit formally using the ε-δ definition. While all countries formally define the derivative, only the Philippines formally defines the definite integral. Given this, it is not surprising that in the findings of Rasslan and Tall's (2002) study, around 63% of students asked to define the definite integral had no answer, and none defined it as the limit of a Riemann sum. These findings suggest that students are more familiar with procedures and notations than with the underlying concepts of integrals. The other countries in this study, which define definite integrals using the Fundamental Theorem of Calculus Part II, tend to teach the results of the theorem rather than the theorem itself (Yoon et al., 2021).

In terms of structure, only the Philippines compartmentalizes calculus as a separate subject. South Africa, New Zealand, and Ireland integrate calculus topics into their broader mathematics curricula. In terms of learning progression, New Zealand adheres to a spiral curriculum. Its Level 7 achievement standards introduce preliminary topics on differentiation and integration, which are revisited with increasing difficulty in Level 8. Related topics are also added to enhance students' competence until they cover the essential concepts of limits, derivatives, and integrals (Wang & McDougall, 2018). Consequently, while the Philippines and New Zealand's Level 8 achievement standards cover the same number of topics, New Zealand students benefit from the prior knowledge acquired in Level 7, paving the way for a smoother learning progression. On the contrary, Filipino students would encounter all 26 calculus topics for the first time over a single semester. However, Filipino teachers report that not all of these 26 topics are taught and not all are covered in depth considering students' inadequate skills and the learning disruptions (Jaudinez, 2019). With this, policymakers in the Philippines could explore adopting a similar spiral approach in designing a high school calculus curriculum to decongest the topics taught in a short span of time.

Meanwhile, considering their limited number of topics covered, South Africa and Ireland's ordinary level curricula offer a slower learning progression. They primarily focus on the foundational concepts in calculus with fewer opportunities to explore more advanced topics like those involving transcendental functions and implicit differentiation. On the other hand, Ireland's higher level has a more rigorous learning progression with the earlier introduction of limits in sequences and series and expanding on topics like differential equations and applications of integrals as students advance.

In light of these findings, university calculus instructors are provided with an overview of the topics covered in high school, highlighting areas that need additional focus and emphasis for a smoother transition. Future research could explore the effect of varying curriculum structures on students' learning of high school calculus. Additionally, it would be valuable to investigate how these intended curricula are implemented in classrooms across these countries. Examining the enacted curriculum could shed light on the educational approaches taken to ensure conceptual understanding in high school calculus which, as the literature reveals, is currently lacking.

Finally, this study calls for the continuous evaluation and refinement of high school calculus curricula to better serve students in their future endeavors, whether in higher education or their pursuit of STEM careers. As Schimdt (2002) aptly states, curriculum matters because what we teach makes a huge difference.

REFERENCES

Ayebo, A., Ukkelberg, S., & Assuah, C. (2016). Success in introductory calculus: The role of high school and pre-calculus preparation. *International Journal of Research in Education and Science*, *3*(25311), 11–19. https://www.ijres.net/index.php/ijres/article/view/131/95

Balmaceda, J. M., Vergara, T. H., De Lara, M. L., Sato, R., & Borja, G. J. (2016). *Basic Calculus Learner's Material* (1st ed.). Department of Education.

Bernardo, A., & Garcia, J. A. (2006). School Improvement in a Centralized Educational System: The Case of the Philippine Basic Educational System. In *School Improvement: International Perspectives* (pp. 227–244). essay, Nova Publishers.

© 2024 | ISSN 2546-0730 (Online) | ISSN 1908-8701 (Print)

Bertram, C. A., Mthiyane, C. C., & Naidoo, J. (2021). The tension between curriculum coverage and Quality Learning: The experiences of South African teachers. *International Journal of Educational Development*, 81, 102353. https://doi.org/10.1016/j.ijedudev.2021.102353

Bilbao, P. P., Corpuz, B. B., & Dayagbil, F. (2015). *Curriculum development for teachers*. Lorimar Publishing.

Borji, V., & Farsani, D. (2023). Intended mathematics curriculum in grade 1: A comparative study. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(3). https://doi.org/10.29333/ejmste/12977

Bressoud, D., Ghedamsi, I., Martinez-Luaces, V., & Törner, G. (2016). Teaching and learning of calculus. *ICME-13 Topical Surveys*, 1–37. https://doi.org/10.1007/978-3-319-32975-8_1

Department of Basic Education. (2011). *Curriculum and assessment policy statement grades 10-12: Mathematics.* https://www.education.gov.za/Curriculum/CurriculumAssessmentPolicyStatements(CAPS)/CAPSFET.aspx

Department of Basic Education. (2012). *National Policy Pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grades R - 12* .

Department of Education. (2016). K to 12 Senior High School STEM Specialized Subject – Calculus. https://www.deped.gov.ph/wp-content/uploads/2019/01/Basic-Calculus.pdf

Department of Education. (2019, August 2). *Education*. gov.ie. https://www.gov.ie/en/policy/655184-education/#introduction

Department of Education. (n.d.). *K to 12 Basic Education Curriculum*. Republic of the Philippines. https://www.deped.gov.ph/k-to-12/about/k-to-12-basic-education-curriculum/

Ghedamsi, I., & Lecorre, T. (2021). Transition from high school to university calculus: A study of connection. *ZDM – Mathematics Education*, *53*(3), 563–575. https://doi.org/10.1007/s11858-021-01262-1

Glatthorn, A. A., Boschee, F., Whitehead, B. M., & Boschee, B. F. (2019). *Curriculum leadership: Strategies for development and implementation*. SAGE Publications, Inc.

Greatorex, J., Rushton, N., Coleman, T., Darlington, E. & Elliott, G. (2019). *Towards a method for comparing curricula.* Cambridge Assessment Research Report. Cambridge, UK: Cambridge Assessment.

Hirsch, C. R., & Reys, B. J. (2009). Mathematics curriculum: A vehicle for School Improvement. *ZDM*, *41*(6), 749–761. https://doi.org/10.1007/s11858-009-0218-0

Houang, R., & Schmidt, W. (2008). *TIMSS International Curriculum Analysis and Measuring Educational Opportunities*.

Ines, J. (2023, December 7). *PISA result indicates PH education system is 5 to 6 years behind – DepEd*. RAPPLER. https://www.rappler.com/philippines/deped-reaction-statement-program-international-student-assessment-result-2022/

Jaschik, S. (2022, February 13). *Does calculus count for too much in admissions?*. Inside Higher Ed | Higher Education News, Events and Jobs. https://www.insidehighered.com/admissions/article/2022/02/14/does-calculus-count-too-much-admissions

Jaudinez, A. S. (2019). Teaching Senior High School Mathematics: Problems and Interventions. *Pedagogical Research*, 4(2), em0031. https://doi.org/10.29333/pr/5779

Jenkin, A., van Zyl, M., & Kannemeyer, L. (n.d.). *Everything Maths: Grade 12 Mathematics*. Siyavula Education.

Keating, M., McElroy, J., Mulvany, D., Murphy, O., O'Loughlin, J., & Townsend, C. (2017). *Active Maths 3* (2nd ed.). Folens Publishers.

Keating, M., McElroy, J., Mulvany, D., O'Loughlin, J., & Townsend, C. (2016). *Active Maths 4* (2nd ed.). Folens Publishers.

Korea Institute for Curriculum and Evaluation, & New Zealand Qualifications Authority. (2015). (rep.). *Comparison of Senior Secondary School Qualifications*.

Knudson, K. (2015, November 20). *The rush to calculus is bad for students and their futures in STEM*. The Conversation. https://theconversation.com/the-rush-to-calculus-is-bad-for-students-and-their-futures-in-stem-49746

Lakeland, R., & Nugent, C. (n.d.-a). *EAS 2.7: Year 12 Mathematics and Statistics*. Nulake Ltd.

Lakeland, R., & Nugent, C. (n.d.-b). *Year 13 EAS Calculus Workbook*. Nulake Ltd.

Macha, W., & Kadakia, A. (2021, September 1). *Education in South Africa*. World Education News + Reviews. https://wenr.wes.org/2017/05/education-south-africa

Ministry of Education. (2014, April 3). *Achievement Objectives for Mathematics and Statistics*. The New Zealand Curriculum Online. https://nzcurriculum.tki.org.nz/The-New-Zealand-Curriculum/Mathematics-and-statistics/Achievement-objectives

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science*. International Association for the Evaluation of Educational Achievement (IEA).

Mullis, I. V. S., Martin, M. O., Goh, S., & Cotter, K. (Eds.) (2016). TIMSS 2015 Encyclopedia: Education Policy and Curriculum in Mathematics and Science. Boston College, TIMSS & PIRLS International Study Center. http://timssandpirls.bc.edu/timss2015/encyclopedia/

National Council for Curriculum and Assessment. (2015). *Mathematics syllabus: Foundation, Ordinary & Higher Level*. https://www.curriculumonline.ie/senior-cycle/senior-cycle-subjects/mathematics/

National Council for Curriculum and Assessment. (2024). *Curriculum*. curriculum online. https://www.curriculumonline.ie/senior-cycle/curriculum/

New Zealand Qualifications Authority, & Korea Institute for Curriculum and Evaluation. (2015). (rep.). *Comparison of Senior Secondary School Qualifications*.

OECD (2023), PISA 2022 Results (Volume I): The State of Learning and Equity in Education, PISA, OECD Publishing, Paris. https://doi.org/10.1787/53f23881-en

O'Meara, N., Prendergast, M., & Treacy, P. (2023). Mathematics in Ireland's upper secondary schools: Why do students choose higher level maths?. *Issues in Educational Research*, *33*(1), 227–246. http://www.iier.org.au/iier33/omeara.pdf

Philippine Business for Education. (n.d.). *Towards greater decentralization of Philippine Basic Education Governance*. PBEd. https://pbed.ph/blogs/48/PBEd/Towards%20Greater%20Decentralization%20of%20Philippine%20Basic%20Education%20Governance

Rasmussen, C., Marrongelle, K., & Borba, M. C. (2014). Research on calculus: What do we know and where do we need to go? *ZDM*, *46*(4), 507–515. https://doi.org/10.1007/s11858-014-0615-x

Rasslan, S., & Tall, D. (2002). Definitions and images for the definite integral concept. In A. Cockburn & E. Nardi (Eds.), Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education, Norwich, UK.

Sadler, P., & Sonnert, G. (2018). The path to college calculus: The impact of high school mathematics coursework. *Journal for Research in Mathematics Education*, *49*(3), 292–329. https://doi.org/10.5951/jresematheduc.49.3.0292

Schmidt, W. H., McKnight, C., Houang, R. T., Wang, H., Wiley, D. E., Cogan, L. S., & Wolfe, R. G. (2001). *Why schools matter: A cross-national comparison of curriculum and learning*. Jossey-Bass Publishers.

Schmidt, W.H., Houang, R.T., & Cogan, L.S. (2002). A Coherent Curriculum: The Case of Mathematics. *The American Educator, 26*, 10.

Thomas, M. O. J., de Freitas Druck, I., Huillet, D., Ju, M. K., Nardi, E., Rasmussen, C., & Xie, J. (2015). Key mathematical concepts in the transition from secondary school to university. In S. J. Cho (Ed.), The proceedings of the 12th international congress on mathematical education (pp. 265–284). Springer, Cham

Törner, G., Potari, D., & Zachariades, T. (2014). Calculus in European classrooms: Curriculum and teaching in different educational and cultural contexts. *ZDM*, 46(4), 549–560. https://doi.org/10.1007/s11858-014-0612-0

Tyler, R. W. (1957). The curriculum then and now. In *Proceedings of the 1956 Invitational Conference on Testing Problems*. Princeton, NJ: Educational Testing Service.

Wang, Z., & McDougall, D. (2018). Curriculum matters: What we teach and what students gain. *International Journal of Science and Mathematics Education*, *17*(6), 1129–1149. https://doi.org/10.1007/s10763-018-9915-x

Yoon, H., Bae, Y., Lim, W., & Kwon, O. N. (2021). A story of the national calculus curriculum: how culture, research, and policy compete and compromise in shaping the calculus curriculum in South Korea. *ZDM – Mathematics Education*, *53*(3), 663–677. https://doi.org/10.1007/s11858-020-01219-w

Zuccheri, L., Zudini, V. (2014). History of Teaching Calculus. In: Karp, A., Schubring, G. (eds) Handbook on the History of Mathematics Education. Springer, New York, NY. https://doi.org/10.1007/978-1-4614-9155-2_24

ABOUT THE AUTHOR

Harold B. Badilla is an Assistant Professor in the Mathematics Department of the University of the Philippines Integrated School (UPIS). He holds bachelor's and master's degrees in mathematics and is currently pursuing a doctorate in data science. Prior to teaching at UPIS, he taught undergraduate mathematics courses at Ateneo de Naga University, handling both general education and major mathematics courses. His research interests include curriculum studies in mathematics and the application of data science in education.